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Laminar free convection in a vertical tube with temperature-dependent viscosity

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Abstract—The role of temperature-dependent viscosity was studied in laminar free convection in a vertical tube. This was carried out by applying a finite-difference method to the governing equations of developing flow inside a vertical tube with an isothermal wall. The only parameter necessary to take the temperature dependence of viscosity into account is the ratio of viscosities at the wall and ambient temperatures. An illustrative example is given to show the discrepancy which may arise when a fluid of highly temperature-dependent viscosity such as glycerin is treated as temperature independent.

INTRODUCTION

LAMINAR flow heat transfer has been of engineering interest for many years because of its wide applications in such devices as heat exchangers. Many investigations have been carried out on laminar flow of gases and liquids whose properties are temperature dependent. When a system with large temperature differences is considered, the thermophysical properties such as viscosity, density, specific heat, coefficient of thermal expansion, and thermal conductivity must be taken as temperature dependent in order to properly understand the physical phenomena. Sparrow and Gregg [1] have studied natural convection of gases and liquid mercury along a vertical flat plate and calculated the appropriate reference temperature so that constant-property results can be applied to variable-property situations. More recently, Carey and Mollendorf [2] have presented a perturbation analysis for natural convection flows in liquids having a temperature-dependent viscosity in freely rising plumes.

Heat transfer in a circular pipe also has been studied extensively. However, many investigations dealt with either forced convection in a horizontal tube [3–5] or combined free and forced convection through a vertical tube [6, 7].

In this study, natural convection inside a vertical tube having an isothermal wall is considered to clarify the role of temperature-dependent viscosity effects. Since in many cases, for high Prandtl number liquids, the variation of properties other than viscosity is negligible, the assumption of only variable viscosity is realistic and applicable to many liquids such as petroleum oils, glycerine, glycols, silicone fluids and molten salts [8]. The present calculations consider the development of both the velocity and temperature profiles of the flow and predict the overall mass flow rate through the tube and the average heat transfer.

ANALYSIS

Consider a steady laminar flow through a vertical tube whose wall temperature is constant and higher than the ambient fluid temperature (see Fig. 1). The buoyancy forces cause the flow to advance vertically in the upward direction inside the pipe.

The nondimensional governing equations are the following:

continuity

$$\frac{\partial u^+}{\partial z^+} + \frac{1}{r^+}\frac{\partial}{\partial r^+}(v^+ r^+) = 0; \tag{1}$$

momentum

$$u^+\frac{\partial u^+}{\partial z^+} + v^+\frac{\partial u^+}{\partial r^+} = -\frac{\mathrm{d}p'^+}{\mathrm{d}z^+} \frac{1}{r^+}\frac{\partial}{\partial r^+}\left(v^+ r^+ \frac{\partial u^+}{\partial r^+}\right) + Gr_0\, T^+; \tag{2}$$

energy

$$u^+\frac{\partial T^+}{\partial z^+} + v^+\frac{\partial T^+}{\partial r^+} = \frac{1}{Pr_0}\frac{1}{r^+}\frac{\partial}{\partial r^+}\left(r^+\frac{\partial T^+}{\partial r^+}\right); \tag{3}$$

continuity in integral form

$$\frac{1}{2}u_0^+ = \int_0^1 r^+ u^+\,\mathrm{d}r^+; \tag{4}$$

where the dimensionless notations below were utilized (subscript 0 refers to any property evaluated at ambient

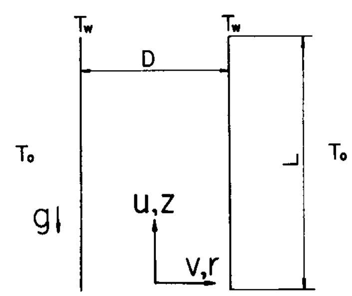

FIG. 1. Model for free convection in a vertical tube.

NOMENCLATURE

C_p	specific heat
D	diameter of circular pipe
Gr_0	Grashoff number
$\bar{h}$	average heat transfer coefficient
k	thermal conductivity
L	length of circular pipe
Nu_D	Nusselt number
p'^+	dimensionless pressure
Pr_0	Prandtl number
Q_{total}	total heat transfer
r	radial coordinate
r^+, z^+	dimensionless coordinates
R	radius of circular pipe
Ra_0	Rayleigh number
T^+	dimensionless temperature
T_0	ambient temperature
T_w	wall temperature
u	longitudinal velocity
u^+, v^+	dimensionless velocities
u_0	inlet velocity
u_0^+	dimensionless inlet velocity
u^*	reference velocity
v	radial velocity
z	longitudinal coordinate.

Greek symbols

β	coefficient of thermal expansion
ν	kinematic viscosity
ν^+	dimensionless kinematic viscosity
ρ	density.

Subscripts

w	properties evaluated at the wall temperature
0	properties evaluated at the ambient temperature.

temperature)

$$u^+ = \frac{u}{u^*}\frac{D/2}{L} \quad \text{where} \quad u^* = \frac{\nu_0}{D/2}, \quad u_0^+ = \frac{u_0}{u^*}$$

$$v^+ = \frac{v}{u^*}$$

$$z^+ = \frac{z}{L}$$

$$r^+ = \frac{r}{D/2}$$

$$T^+ = \frac{T-T_0}{T_w-T_0}$$

$$p'^+ = \frac{(D/2)^2}{\rho L^2 u^{*2} Gr^2} p' \quad \text{where}$$

$$p' = p - p_0 + \rho_0 g x$$

$$Gr = \frac{g\beta_0(T_w-T_0)(D/2)^2}{u^{*2}L}$$

$$\nu^+ = \frac{\nu}{\nu_0}.$$

The boundary conditions in equations (1)–(3) are

$$u^+ = u_0^+, \quad v^+ = 0 \quad \text{and} \quad T^+ = 0 \quad \text{at} \quad z^+ = 0 \quad \text{and} \quad 0 \leqslant r^+ < 1$$

$$u^+ = 0, \quad v^+ = 0 \quad \text{and} \quad T^+ = 1 \quad \text{at} \quad r^+ = 1$$

$$\frac{\partial u^+}{\partial r^+} = 0, \quad \frac{\partial T^+}{\partial r^+} = 0 \quad \text{and} \quad v^+ = 0 \quad \text{at} \quad r^+ = 0$$

$$p'^+ = 0 \quad \text{at} \quad z^+ = 0 \quad \text{and} \quad 1. \qquad (5)$$

It is necessary, before proceeding, to choose a particular viscosity–temperature relation. The one adopted was a compromise between simplicity and satisfactory agreement with experimental data. Exponential relations, which are discussed in detail in refs. [5, 6, 9] seemed most appropriate to fulfil these conditions and the following was adopted

$$\nu^+ = \exp[T^+ \ln \nu_w^+]. \qquad (6)$$

This equation has the advantage of agreeing with the known viscosities at both the wall and ambient temperatures. That the representation is satisfactory over the temperature ranges considered here is shown in Fig. 2 which compares the viscosity–temperature equation with data reported for glycerin in ref. [11].

Some of the characteristics of this particular problem include:

(1) The average velocity cannot be assumed known for free convection where the temperature difference $\Delta T = T_w - T_0$ creates the flow. Thus, the average velocity depends on the Grashof number and must be determined.

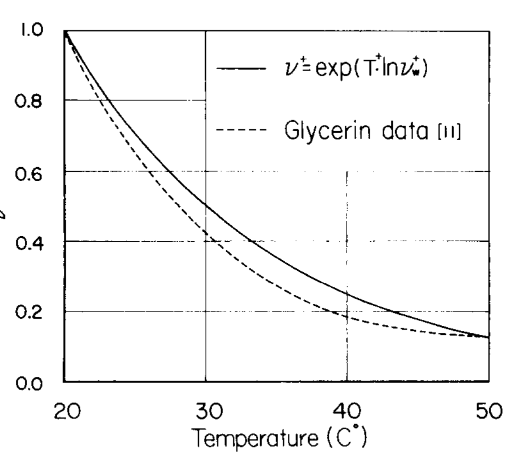

FIG. 2. Comparison of experimental data with viscosity–temperature relation ($T_0 = 20°C$, $T_w = 50°C$).

(2) Since both ends of the pipe are open to the ambient fluid and the pressure difference is only hydrostatic, this imposes the boundary condition that the dimensionless pressure, p'^+, is zero at both ends.

(3) Since an exponential type correlation was adopted to express the viscosity variation with temperature, i.e. $v^+ = \exp[T^+ \ln v_w^+]$, the only new parameter as compared to the constant property solution is the viscosity ratio, $v_w^+ = v_w/v_0$.

The following is the definition of the average heat transfer coefficient

$$Q_{\text{total}} = \bar{h}A(T_w - T_0) = \bar{h}\cdot 2\pi RL(T_w - T_0) \tag{7}$$

also

$$Q_{\text{total}} = \rho c_p \int_0^R u(T - T_0) 2\pi r \, dr. \tag{8}$$

By nondimensionalizing equation (8), one obtains

$$\frac{Q_{\text{total}}}{\rho c_p u^* L(T_w - T_0)\pi D} = \int_0^1 u^+ T^+ r^+ \, dr^+. \tag{9}$$

Substituting equation (7) into equation (9)

$$\frac{\bar{h}R}{\rho c_p v_0} = \int_0^1 u^+ T^+ r^+ \, dr^+. \tag{10}$$

The average Nusselt number can be found from the average heat transfer coefficient, $\bar{h}$

$$\overline{Nu_D} = \frac{\bar{h}D}{k} = 2Pr \int_0^1 u^+ T^+ r^+ \, dr^+. \tag{11}$$

Fully developed flow region

One has to be quite careful in defining fully developed flow in free convection in a vertical tube with variable properties. Normally with variable properties, there is no such thing as fully developed flow since if the viscosity changes in the flow direction, both the velocity and temperature fields will always be a function of the flow direction coordinate. With the thermal boundary condition of constant wall temperature, however, free convection flow does become fully developed because the buoyancy forces cause the asymptotic fluid temperature to become identical to the wall temperature and thus there are no longer any variable property effects in the cross-section. (This situation will not be attained with a constant heat flux boundary condition where fully developed flow cannot occur.)

Once the fluid temperature becomes constant, there are no longer any differential body forces in the cross-section and the velocity profile will assume a constant shape driven by the constant temperature difference between the wall and ambient temperatures. Unlike forced convection with a constant property fluid in a horizontal tube however, this shape is sensitive to the length of the entrance region. Thus, it is possible to define two types of fully developed flow:

(1) When there is an appreciable entrance region but finally the boundary layers meet within the duct. The velocity field for this situation can only be obtained by solving the complete equations [equations (1)–(4)] because of variable property effects. This will be called fully developed flow of the first kind (FDF-I).

(2) When the entrance region is vanishingly small (low Grashof and Prandtl numbers) the buoyancy forces on all fluid elements are constant and there are no variable property effects. This will be called fully developed flow of the second kind (FDF-II).

This fully developed flow of the second kind can be analyzed in a simple way in order to check the asymptotic solutions of the developing flow. Since by definition the pressure gradient term in the momentum equation, equation (5), may be assumed to be zero, one obtains the following expression

$$\frac{1}{r^+}\frac{\partial}{\partial r^+}\left(v_w^+ r^+ \frac{\partial u^+}{\partial r^+}\right) + Gr_0 = 0. \tag{12}$$

Thus, the velocity profile can be found by integrating twice

$$u^+ = \frac{1}{4}\frac{Gr_0}{v_w^+}(1 - r^{+2}). \tag{13}$$

Substituting equation (13) into equation (4)

$$u_0^+ = \frac{1}{8}\frac{Gr_0}{v_w^+}. \tag{14}$$

For fully developed flow, the average Nusselt number in equation (11) may be expressed as

$$\overline{Nu_D} = Pr_0 u_0^+ = \frac{1}{8}\frac{Ra_0}{v_w^+}$$

where

$$Ra_0 = Pr_0 Gr_0. \tag{15}$$

Developing flow region

A finite-difference procedure was used in the developing region of the channel flow. The governing equations, written in finite-difference form are:

continuity

$$\frac{u_{j+1,k+1}^+ - u_{j,k+1}^+ + u_{j+1,k}^+ - u_{j,k}^+}{2(\Delta z^+)} + \frac{1}{r_k^+}\frac{r_{k+1}^+ u_{j+1,k+1}^+ - r_k^+ u_{j+1,k}^+}{\Delta r^+} = 0; \tag{16}$$

momentum

$$u_{j,k}^+ \frac{u_{j+1,k}^+ - u_{j,k}^+}{\Delta z^+} + v_{j,k}^+ \frac{u_{j+1,k+1}^+ - u_{j+1,k-1}^+}{2(\Delta r^+)}$$
$$= -\frac{p_{j+1}^+ - p_j^+}{\Delta z^+} + Gr\,T_{j+1,k}^+ + \frac{v_{j+1,k}^+}{r_k^+}\frac{u_{j+1,k+1}^+ - u_{j+1,k-1}^+}{2(\Delta r^+)}$$
$$+ \frac{1}{2}\frac{[v^+u^+]_{j+1,k+1} - 2[v^+u^+]_{j+1,k} + [v^+u^+]_{j+1,k-1}}{(\Delta r^+)^2}$$
$$- \frac{1}{2}u_{j+1,k}^+ \frac{v_{j+1,k+1}^+ - 2v_{j+1,k}^+ + v_{j+1,k-1}^+}{(\Delta r^+)^2}$$
$$+ \frac{1}{2}v_{j+1,k}^+ \frac{u_{j+1,k+1}^+ - 2u_{j+1,k}^+ + u_{j+1,k-1}^+}{(\Delta r^+)^2}; \tag{17}$$

energy

$$u^+_{j,k}\frac{T^+_{j+1,k}-T^+_{j,k}}{\Delta z^+}+u^+_{j,k}\frac{T^+_{j+1,k+1}-T^+_{j+1,k-1}}{2(\Delta r^+)}$$
$$=\frac{1}{Pr_0}\left[\frac{1}{r^+_k}\frac{T^+_{j+1,k+1}-T^+_{j+1,k-1}}{2(\Delta r^+)}+\frac{T^+_{j+1,k+1}-2T^+_{j+1,k}+T^+_{j+1,k-1}}{(\Delta r^+)^2}\right]; \quad (18)$$

continuity in integral form (using Simpson's rule)

$$\frac{\Delta r^+}{3}[u^+_{j+1,1}r^+_1+4u^+_{j+1,2}r^+_2+2u^+_{j+1,3}r^+_3+\cdots+2u^+_{j+1,n-1}r^+_{n-1}+4u^+_{j+1,n}r^+_n+u^+_{j+1,n+1}r^+_{n+1}]=\frac{1}{2}u^+_0. \quad (19)$$

The finite-difference equations were programmed along with the boundary conditions. Only a half channel was considered due to geometric symmetry. The mesh size was chosen to be $\Delta r^+ = 0.01$ and Δz^+ depends on the location, i.e. for the rapidly developing flow region very close to the inlet a very fine mesh was chosen and for the nearly developed flow region a relatively rough mesh was used.

At a given axial location, a set of 201 simultaneous equations was constructed and solved for the axial velocities and temperatures as well as the pressure drop and local Nusselt number after the values of Pr_0 and Gr_0 were selected and the inlet velocity guessed. Since the matrix is almost tridiagonal, matrix inversion was carried out by a scheme discussed in ref. [9]. Such a scheme yielded a considerable reduction in computational time for the fine mesh regions. After solving this set, the velocities in the transverse direction were calculated by substituting the axial velocities into the continuity equation. With velocities and temperatures computed in each mesh, the above procedure was continued or 'marched' in the flow direction until the end of the channel. This was repeated three times until $p^{+\prime}$ was sufficiently close to zero at the end of the channel.

It was also necessary to carry out a downstream iteration with regard to the viscosity and the temperature and velocity profiles. Since at any downstream station, the temperature distribution at the next station is unknown, the viscosity at this next station is also unknown. The iteration involved using an upstream temperature-dependent viscosity to determine the next temperature distribution and then to use this temperature distribution to re-evaluate the viscosity, etc.

DISCUSSION OF RESULTS

The computational results are shown in Figs. 3(a)–(e) for different Prandtl numbers with the Grashof number as the independent variable. Fully developed flow solutions (FDF-II) for the inlet velocity, u^+_0, and the average Nusselt numbers were compared with the finite-difference calculations at low Grashof numbers

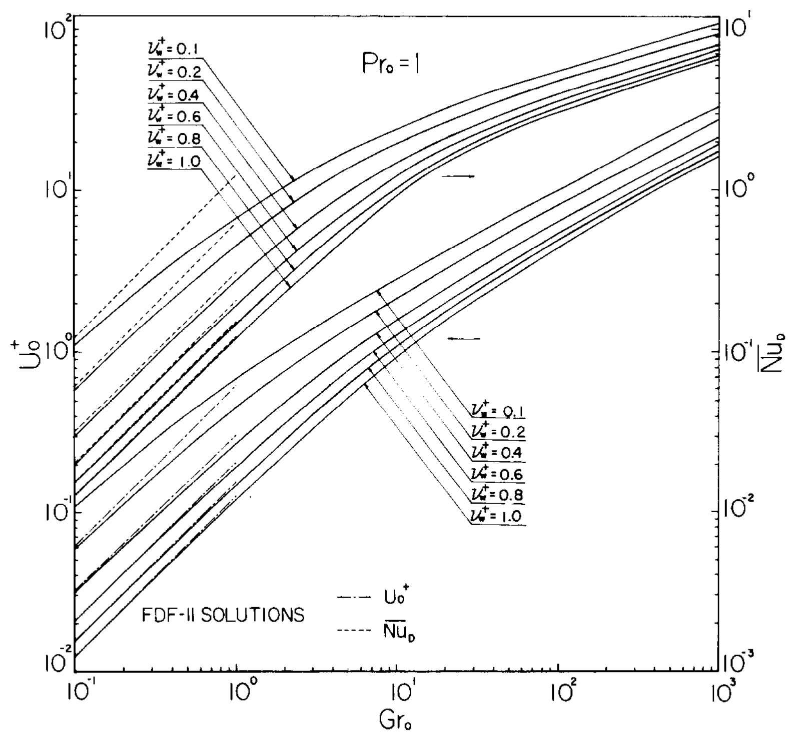

FIG. 3(a). Variation of dimensionless average velocity and average Nusselt number with Grashof number with various parameters at $Pr_0 = 1$. Dashed lines indicate FDF-II solutions.

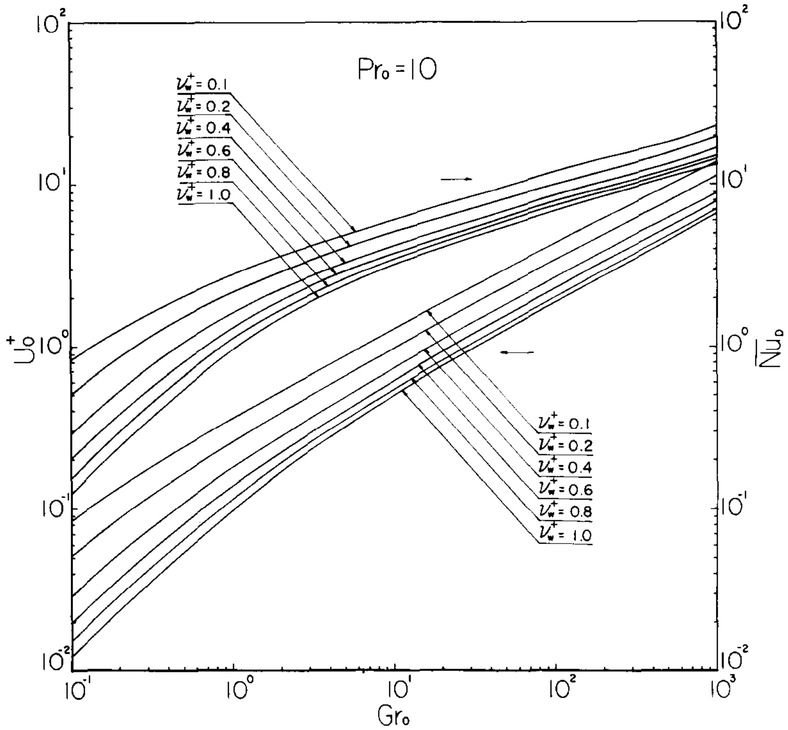

FIG. 3(b). Variation of dimensionless average velocity and average Nusselt number with Grashof number with various viscosity parameters at $Pr_0 = 10$.

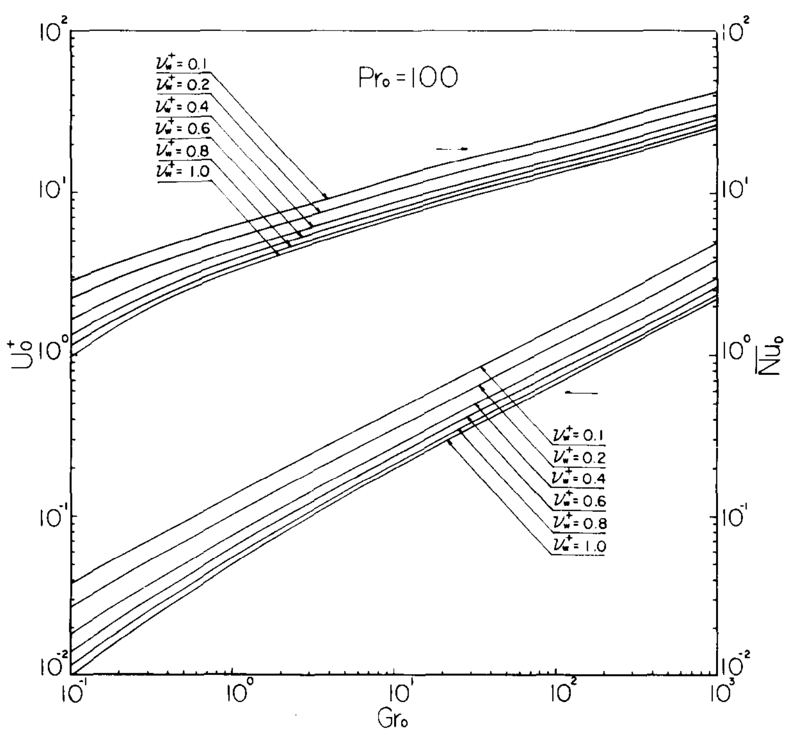

FIG. 3(c). Variation of dimensionless average velocity and average Nusselt number with Grashof number with various viscosity parameters at $Pr_0 = 100$.

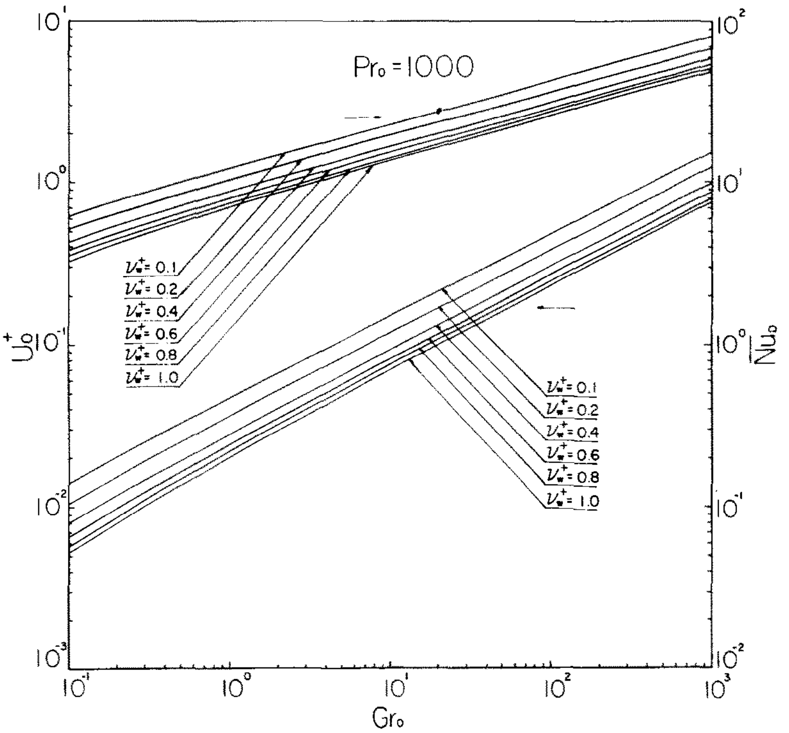

FIG. 3(d). Variation of dimensionless average velocity and average Nusselt number with Grashof number with various viscosity parameters at $Pr_0 = 1000$.

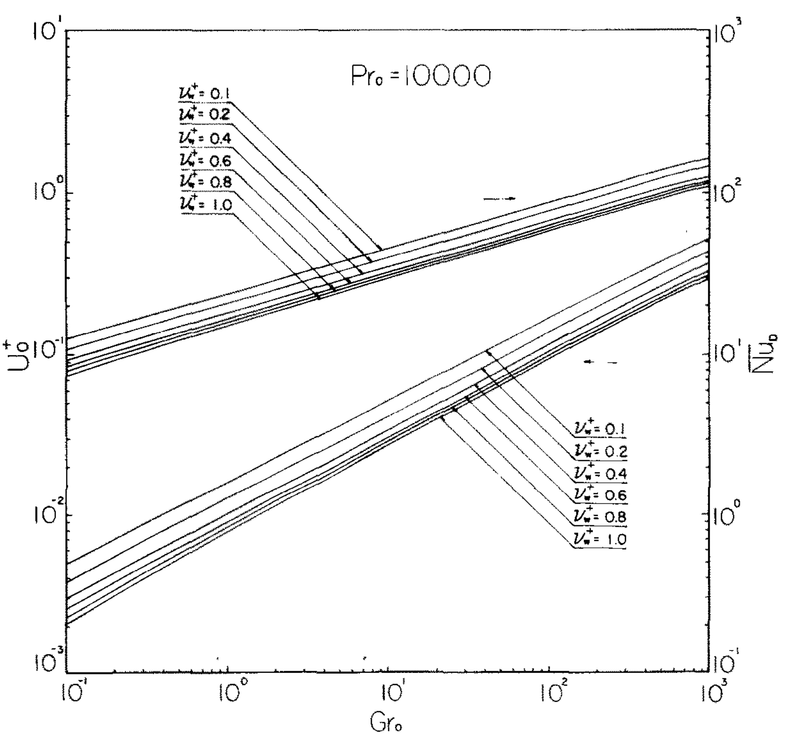

FIG. 3(e). Variation of dimensionless average velocity and average Nusselt number with Grashof number with various viscosity parameters at $Pr_0 = 10000$.

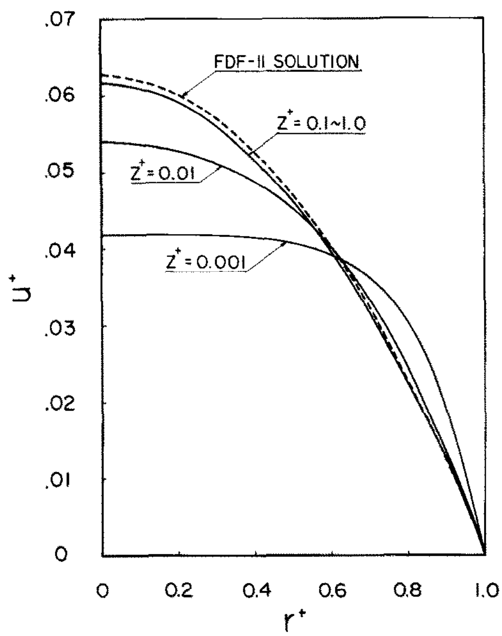

FIG. 4. Velocity profiles at various axial locations with $Gr_0 = 0.1$, $Pr_0 = 1.0$, and $\nu_w^+ = 0.4$. Entrance region is short. Dashed line is FDF-II solution.

$Gr = 0.1 \sim 0.3$ and low Prandtl number $Pr = 1$. Excellent agreement can be seen in Fig. 3(a) except for values of the viscosity parameter of $\nu_w^+ = 0.1$ and 0.2. The slight departure for these two cases are due to the relatively short length of the fully developed flow region over the entire channel as discussed previously.

Another qualitative check on the computational results is that for very large Prandtl numbers the thermal boundary layer is confined close to the wall so that the effects of variable viscosity on the main flow are suppressed. The graphs show such a tendency for high Prandtl numbers.

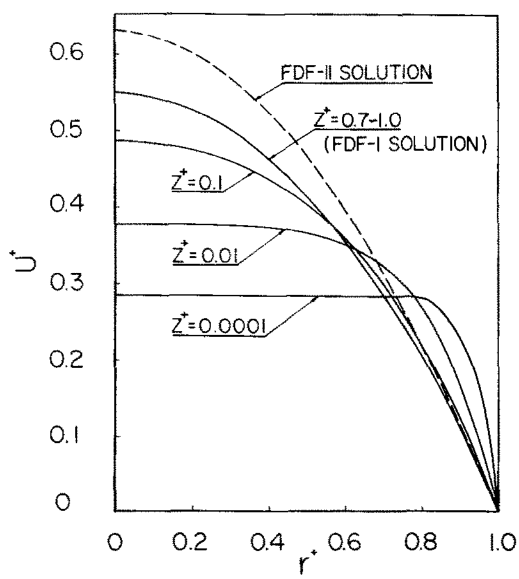

FIG. 5. Velocity profiles at various axial locations with $Gr_0 = 1.0$, $Pr_0 = 1.0$, and $\nu_w^+ = 0.4$. Fully developed flow of the first kind exists. Dashed line is FDF-II solution.

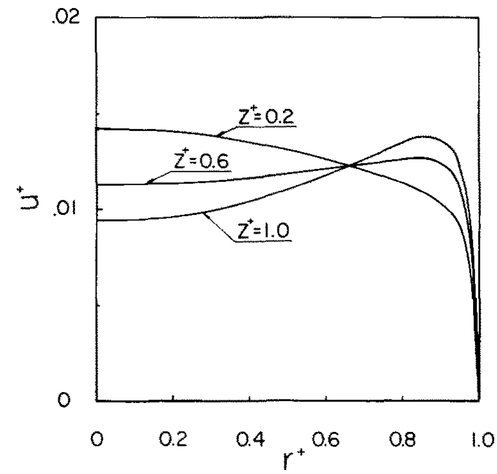

FIG. 6. Velocity profiles at various axial locations at $Gr_0 = 0.66$, $Pr_0 = 10^4$, and $\nu_w^+ = 0.127$. No type of fully developed flow exists.

Figures 4–6 show velocity profiles at different axial locations under various flow conditions. Figure 4 shows the development of the velocity field at a low Rayleigh number where the entrance length is relatively small ($z^+ \simeq 0.1$). Thus the velocity profile approaches the FDF-II solution as shown by the dashed line in the figure. Figure 5 shows the situation at a moderate Rayleigh number where fully developed flow of the first kind occurs. The dashed line in the figure is for FDF-II which differs significantly from FDF-I. Figure 6 shows the velocity field development at a large Rayleigh number. In this case, the wall boundary layers do not meet and neither FDF-I or FDF-II occurs.

ILLUSTRATIVE EXAMPLE

Variable viscosity effects can be illustrated by a numerical example using physical dimensions. The fluid chosen here is similar to glycerin in its properties, but the value of Pr_0 was arbitrarily chosen as 10^4 so that the solution could be obtained from Fig. 3(e). The purpose of the example is to indicate quantitatively the different predictions for average velocity and average Nusselt number when variable property effects are considered or not.

The calculations were performed assuming a 10 cm diameter, 1 m long circular pipe ($L/D = 10$), a wall temperature of 50°C and an ambient temperature of 20°C. Using the physical properties of glycerin [11] (except for the Prandtl number as discussed above), the relevant properties are:

$$\nu_0 = 1.18 \times 10^{-3}\,\mathrm{m^2\,s^{-1}} \quad \beta = 0.50 \times 10^{-3}\,\mathrm{K^{-1}}$$

$$\nu_w = 1.50 \times 10^{-4}\,\mathrm{m^2\,s^{-1}}$$

$$k = 0.286\ \mathrm{W\,m^{-1}\,K^{-1}}$$

and since the viscosity parameter is the ratio of ν_w/ν_0

$$\nu_w^+ = 0.127.$$

Thus the parameters Gr_0 and Pr_0 and the reference velocity are

$$Pr_0 = 10^4$$

$$Gr_0 = \frac{g\beta(T_w - T_0)R^2}{u^{*2}L} = 0.660$$

$$u^* = v_0/R = 2.36 \times 10^{-2}\,\mathrm{m\,s^{-1}}.$$

The dimensionless average velocity and the heat transfer coefficient from Fig. 3(e) are:

$$u_0^+ = \frac{u}{u^*}\frac{R}{L} = 0.0115: \quad u_0 = 0.543\,\mathrm{cm\,s^{-1}}$$

$$\overline{Nu_D} = \frac{\bar{h}D}{k} = 19.4: \quad \bar{h} = 55.6\,\mathrm{W\,m^{-2}\,K^{-1}}$$

and the total heat transfer from the tube is

$$Q_{\text{total}} = \bar{h}A(T_w - T_0) = 534\,\mathrm{W}.$$

If one now repeats the calculation assuming constant properties evaluated at the ambient temperature with $v_w^+ = 1$ which is the constant property solution

$$u_0^+ = 0.0058: \quad u_0 = 0.274\,\mathrm{cm\,s^{-1}}$$

$$\overline{Nu_D} = 12.5: \quad \bar{h} = 35.9\,\mathrm{W\,m^{-2}\,K^{-1}}$$

$$Q_{\text{total}} = 338\,\mathrm{W}.$$

Thus, the consideration of variable property effects increases the predicted average velocity by 98% and the total heat transfer by 60%.

SUMMARY AND CONCLUSIONS

Natural convection inside a vertical tube with an isothermal wall was investigated including a consideration of temperature-dependent fluid viscosities. A numerical solution was obtained from the conservation equations for the average heat transfer coefficient and the average flow velocity. An asymptotic solution was also derived for the case when the entrance length is vanishingly small.

Because of the constant wall temperature condition, two types of fully developed flow can be identified, one when the entrance length is vanishingly small and another when the entrance length is finite but less than the tube height.

A numerical example is presented which illustrates that the consideration of temperature-dependent viscosity has a very significant effect on the predicted average velocity and heat transfer as compared to constant property predictions.

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CONVECTION LIBRE LAMINAIRE DANS UN TUBE VERTICAL AVEC UNE VISCOSITE VARIANT AVEC LA TEMPERATURE

Résumé—On étudie le rôle de la viscosité dépendant de la température sur la convection libre dans un tube vertical. On applique une méthode aux différences finies sur les équations de l'écoulement qui se développe à l'intérieur d'un tube vertical à paroi isotherme. Le seul paramètre nécessaire pour traduire la dèpendance à la température de la viscosité est le rapport des viscosités à la température de la paroi et à l'ambiance. Une illustration est donnée pour montrer l'écart qui peut exister lorsqu' un fluide à viscosité fortement variable en fonction de la température, comme la glycérine, est traité comme sans dépendance à la température.

LAMINARE FREIE KONVEKTION IM SENKRECHTEN ROHR BEI TEMPERATURABHÄNGIGER VISKOSITÄT

Zusammenfassung—Der Einfluß der temperaturabhängigen Viskosität wurde bei laminarer freier Konvektion in einem senkrechten Rohr untersucht, indem die Bilanzgleichungen der sich ausbildenden Strömung bei isothermen Wänden mit einem Differenzenverfahren gelöst wurden. Der einzige Parameter, der notwendig ist, um die Temperaturabhängigkeit der Viskosität zu berücksichtigen, ist das Verhältnis der Viskositäten bei den Temperaturen der Wand und der angrenzenden Flüssigkeit. Ein anschauliches Beispiel zeigt die Diskrepanz, die auftreten kann, wenn ein Fluid mit stark temperaturabhängiger Viskosität wie z.B. Glyzerin als temperaturunabhängig behandelt wird.

ЛАМИНАРНАЯ СВОБОДНАЯ КОНВЕКЦИЯ В ВЕРТИКАЛЬНОЙ ТРУБЕ ПРИ УЧЕТЕ ЗАВИСИМОСТИ ВЯЗКОСТИ ОТ ТЕМПЕРАТУРЫ

Аннотация—Исследовалась роль зависимости вязкости от температуры при ламинарной свободной конвекции в вертикальной трубе. Для решения основных уравнений, описывающих течение внутри вертикальной трубы с изотермической стенкой, использовался метод конечных разностей. В результате учета температурной зависимости вязкости появляется параметр в виде отношения вязкостей при температурах стенки и окружающей среды. Иллюстрируются возможные расхождения в случае, когда жидкость, вязкость которой сильно зависит от температуры, например, глицерин, рассматривается как жидкость с независящей от температуры вязкостью.